Quantum Signature of Cosmological Large Scale Structures

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We demonstrate that to all large scale cosmological structures where gravitation is the only overall relevant interaction assembling the system (*e.g.* galaxies), there is associated a characteristic unit of action per particle whose order of magnitude coincides with the Planck action constant h. This result extends the class of physical systems for which quantum coherence can act on macroscopic scales (as *e.g.* in superconductivity) and agrees with the absence of screening mechanisms for the gravitational forces, as predicted by some renormalizable quantum field theories of gravity. It also seems to support those lines of thought invoking that large scale structures in the Universe should be connected to quantum primordial perturbations as requested by inflation, that the Newton constant should vary with time and distance and, finally, that gravity should be considered as an effective interaction induced by quantization.

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I. INTRODUCTION

Explaining the large scale structure of the Universe is one of the hardest task of modern cosmology since the growing amount of observations seems to escape any coherent scheme able to connect all the parts of the puzzle.

Essentially, from the fundamental physics point of view, we would like to reconduct cosmic structures and their evolution to some unifying theory in which all the today observed interactions are treated under the same standard. In this case, what we observe nowaday on macroscopic, astrophysical scales would be just a consequence of quantum fluctuations at early epochs. Then, we should seek for some "enlarging" mechanism which after one (or more than one) symmetry breaking would be capable of yielding structures like galaxies from primordial quantum spectra of perturbations.

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The so called "inflationary paradigm" [1] related to the several unifying theories (*e.g.* superstrings, GUT, SUSY, and so on) should be succesful if some "experimentum crucis" would select the right model.

On the other hand, particle physicists need cosmological predictions and observations since the energies for testing unified theories are so high that it is extremely unlikely they will be ever reached on earth–based laboratories.

As a matter of fact, cosmology needs particle physics and vice versa. The point is that remnants of primordial epochs should be found by cosmological observations and, by them, one should constraint elementary particle physics models.

This philosophy has been pursued by several researchers; first of all by Sakharov [2] who in 1965 argued that quantum primordial fluctuations should have expanded towards the present epoch leading first to classical energy–density perturbations and, after the decoupling from the cosmological background, to the observed galaxies, clusters and superclusters of galaxies. Shortly, the underlying issue of any modern theory of cosmological perturbation is this: primordial quantum fluctuations should be enlarged by cosmological dynamics to the present large scale structures. Now the problem is not only whether observations agree with this scheme (*e.g.* COBE and IRAS data or large scale structure surveys [3]) but, mainly, whether the astrophysical and cosmological systems "remember" their quantum origin or not.

In some sense, the question becomes philosophical: It is well known that quantum and classical domains are distinct; the frontier is marked by the Planck constant h which separates microscopic from macroscopic scales. If the lengths, the energies, the times become much larger than h, we are in the classical physics regime with likely little hope of recovering quantum signatures.

However, despite of this apparent sharp division of the classical and quantum worlds, macroscopic quantum phenomena exist and some behaviors of classical systems can be explained only in the framework of quantum mechanics. The high T_c superconductivity and several other macroscopic coherent systems (*e.g.* optical fibres) are famous instances of these peculiar phenomena in which a quantum "memory" persists at the macroscopic scale.

Recently, a new intriguing conjecture has been proposed to find signatures of h at the classical, macroscopic scales: Francesco Calogero has argued about a possible gravitational origin of quantization emerging thanks to the universal interaction of every particle in the Universe with the gravitational stochastic background field generated by all other particles [4].

In the framework of this fluctuative Machian scheme, Calogero has been able to show how classical nonlinearity and chaoticity of the gravitational interaction in the Universe yields a unit of action per particle that coincides, in order of magnitude, with h.

Further studies [5] have generalized the scheme of Calogero to the other fundamental interactions responsible for large scale macroscopic structures, finding that h is the characteristic action per particle also for macroscopic systems not bound by gravitational interactions but by other forces, *e.g.* electromagnetic.

In this new scheme, classical laws of force $F(R)$ describing the interactions among the constituents of N–particle systems of mean length scale R, lead to h as the characteristic action per particle. The forces $F(R)$ considered can be, for instance, the electromagnetic interactions between charged particles in large macroscopic systems as charged beams in particle accelerators, plasmas, and neutral dipolar crystals, or the strong interactions between quarks in hadronic bound aggregates, and so on. In other words, $F(R)$ needs not necessarily have to be the gravitational interaction.

The conclusion seems to be that the space–time scales of several mesoscopic and macroscopic coherent aggregates are ruled by characteristic actions of order h. From this point of view, gravitation loses the privileged status of "origin" of quantization that it played in the original scheme of Calogero [4], while the central result of his investigation, as generalized by De Martino *et al.* [5] is that it provides a method to find some quantum mechanical signature in classical macroscopic structures, and to connect it to the observed measured values of their space and time scales.

Having such a procedure at hand, it seems then very natural and appealing to investigate whether it can be applied to determine the existence of unambiguous quantum mechanical signatures or "memories" for large scale cosmological structures. This is exactly the issue considered in the present paper: in

particular, to see whether it is possible to explain the large scale cosmological structures by finding h as the characteristic unit of action for systems as galaxies, clusters of galaxies, and super clusters of galaxies.

As we shall see below, several renormalizable quantum theories of gravity require a modification, in the low energy limit, of Newton law. Furthermore, if we do not require enormous amounts of dark matter as the only mechanism to explain the puzzle of the present day astrophysical observations, a scale–dependent gravitational interaction is also needed.

In this framework, we will show that the method by Calogero in the generalized formulation of De Martino *et al.* [5] can be successfully applied to macroscopic systems where the overall relevant interaction is gravitation (that is where the size of the bound systems is determined by gravity alone) and that in all cases the characteristic unit of action per constituent is h. Therefore the original result of Calogero holding for the entire Universe is recovered also for cosmological structures at smaller scales, and their quantum signatures become evident.

The paper is organized as follows. In Sec. 2, we briefly review why varying effective gravitational couplings and non–Newtonian effective potentials can avoid several shortcomings in fundamental elementary physics and cosmology. Sec. 3 is devoted to the discussion of the method of Calogero generalized by De Martino *et al.* to determine the characteristic minimal unit of action per constituent in classical macroscopic systems. In Sec. 4, we determine the characteristic minimal unit of action per constituent in the case of gravitational large scale cosmological structures and find, also in this case, that it is h. A major role is played by the spatial scale of the structure, by the ratio of its mass with the number of baryons present in it, and by the space–time variation of G_N. Conclusions are drawn in Sec.5.

II. THE VARIATION OF G_N AND THE NON–NEWTONIAN EFFECTIVE POTENTIALS

The possibility of considering a variable Newtonian coupling constant is, at least, sixty years old. In 1937 Dirac [6] put forward his so called Large Numbers Hypothesis [7] in which some intriguing numerical coincidences such as that of the ratio of the electromagnetic to gravitational force with the number of protons in the Universe and with the age of the Universe could be explained in the framework of some unified theory of the micro– and macrophysics. In order to keep the constant values of e, the electron charge, m_e, the electron mass, and m_p, the proton mass, Dirac asked for a variation[1] of G of the form

$$G \sim \frac{1}{t}\,. \tag{1}$$

In this hypothesis, the gravitational strength goes to zero for large times. Similar arguments apply also to the cosmological constant and, in the framework of the Large Numbers Hypothesis, it is argued by several authors (see, for instance, [9], [10]) that

$$\Lambda \sim \frac{1}{t^2}\,. \tag{2}$$

A time–dependent gravitational coupling was conceived also by Sciama [11] and Jordan [12] who provided further arguments supporting this view. In the Brans–Dicke approach [13], General Relativity was modified by introducing a scalar field in the equations of motionto to make them consistent with Mach principle. Such a consistency is in fact achieved if the gravitational coupling is a variable quantity.

[1] From now on, $G_N = 6.67 \times 10^{-8}\mathrm{g}^{-1}\mathrm{cm}^3\mathrm{s}^{-2}$ denotes the Newton constant measured by Cavendish–like experiments; $G(r,t)$ denotes a variable gravitational coupling whose possible explicit forms will be given below. In general, such a coupling can depend also on a space–time dependent scalar field $\phi(r,t)$ [8].

More recently, the so–called induced gravity theories [14] have inquired the possibility that the gravitational and cosmological constants may not be phenomenological parameters to be introduced by hand, but they might be rather induced from the spontaneous symmetry breaking of a scalar field ϕ nonminimally coupled to the Ricci scalar $\mathcal{R}$ in the interaction Lagrangian [15]. The resulting gravitational effective action will contain higher–order terms in the geometrical invariants like $\mathcal{R}^2$ or $\mathcal{R}^{\mu\nu}\mathcal{R}_{\mu\nu}$ [16],[17],[18], nonminimally coupled terms like $\xi\phi^2\mathcal{R}$ [14],[19], or nonminimally–coupled–higher–order terms like $\xi\phi^2\mathcal{R}^2$ [20]. Such theories are renormalizable at one–loop level when graviton–graviton or matter–graviton interactions are considered. To achieve such a result, two effective running constants must be renormalized, G_{eff} and Λ_{eff}, which, in the low energy limit, reduce to G_N and Λ.

From a cosmological point of view, this kind of theories can give rise to inflationary behaviours solving the shortcomings of the standard cosmological model [21],[22]. The mechanism giving rise to the induced gravitational interaction resembles that of vacuum polarization in QED, but with some relevant differences. The vacuum energy of such theories is very small but the net effect of quantum fluctuations can provide sizeable corrections at all scales of distances since gravity coherently couples with itself and with any form of matter. Besides, the absence of screening mechanism for gravitons [23] allows that quantum gravity effects can play a role also at macroscopic and cosmological scales [24]. Although these one–loop quantum gravity theories are known to exhibit pathological behaviors with respect to unitarity (ghost poles in the tree–level propagators), their breakdown is expected near the Planck scale (epoch) while larger scales are nonsensitive to this shortcoming. Thus, these theories can be considered the effective theories of gravity valid at length scales much larger than the Planck length.

One of the main results of this approach is that the renormalization group equation for the gravitational coupling can be analyzed in some detail [18],[25],[26], and, depending on the values of the parameters and on the momentum scales considered in studying the behavior of the β functions, G_{eff} increases or decreases with the distance. Then, in general, we can write

$$G(r) = G_N f(r)\,, \tag{3}$$

where G_N, as we have said, is the Newton constant as measured in laboratory.

If we are dealing with higher–order theories of the form [18]

$$\mathcal{A} = \int d^4x\sqrt{-g}\left\{\Lambda - \frac{\mathcal{R}}{16\pi G_N} + c_1\mathcal{R}^2 + c_2\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \cdots\right\}\,, \tag{4}$$

one can show that a consistent choice is

$$G(r) = \chi G_N \left(\frac{r}{r_0}\right)^{\eta} \ln\left(\frac{r}{r_0}\right)\,, \tag{5}$$

where the parameters η, χ, and r_0 depend on the details of the model [27].

Another approach [17] exploits a technique which treats the higher derivative terms as if they incorporated additional massive bosons, yielding Yukawa–like effects in the Newtonian potential in the low energy limit. In this case one has

$$V(r) = -\frac{G(r)M}{r}\,, \tag{6}$$

where $G(r)$ can take the form

$$G(r) = G_N\left(1 + a_0 e^{-r/r_0}\right)\,, \tag{7}$$

or the form

$$G(r) = G_N\left(1 + d_1 e^{-m_1(r)r} + d_2 e^{-m_2(r)r}\right)\,, \tag{8}$$

as shown by Stelle [16].

Some authors [28],[29] have exploited potentials like (7) in order to explain the flat rotation curves of spiral galaxies. Their approach is phenomenological and the adjustments of parameters is often “ad hoc” in order to fit the experimental data.

Yukava–like corrections result also if one deals with galaxies as potential wells [30]. In this case, the theory of Newtonian perturbations on a background fluid is sufficient to explain, in a quite simple way, the dynamics of hot components of galaxies like the bulge. In all these approaches, the amount of dark matter still required to match observations is greatly suppressed compared to the amount needed to “cure” the standard cosmological model.

From the above discussion, it appears not so unnatural to allow for a running gravitational coupling. Such a variation can be with respect to the time (*e.g.* in cosmology) or, in general, with respect to the scale.

In the present paper we show that considering a scale–dependent $G(r)$, it is possible to determine, exploiting the results of Calogero [4] and of De Martino *et al.* [5], a characteristic minimal action of the order of Planck constant h for any system bound only by gravity. This is the case for galaxies, clusters, and superclusters of galaxies (On the other hand, we exclude stars from our analysis, since in this case electromagnetic and nuclear interactions play a crucial role in the binding of the system).

III. THE CHARACTERISTIC UNIT OF ACTION FOR MACROSCOPIC SYSTEMS

As we have seen in the previous section, the issue of a running gravitational coupling is relevant in the context of quantum theories of gravity and quantum cosmology. Here we want to show how to define a characteristic unit of action for the individual consituents of bound, stable dynamical systems of global dimension R, made of N elementary constituents. By this approach, it is possible to find that for stable macroscopic systems such an action is of order h [5]. Our goal is to show that any stable gravitationally bound system, where the overall relevant interaction between constituents is gravity, has exactly h as characteristic unit of action per constituent, if we consider theories of induced gravity with running Newtonian constant. What we mean by “constituents” in the context of cosmology are stars or even galaxies. We will focus on this application in the next section. Let us first illustrate the general procedure, whose detailed derivation appears in [5].

Let $F(r)$ be the modulus of a classical law of force (overall attractive) acting on the N elementary components of mass m which constitute a macroscopic physical system. Let v be the some mean local characteristic velocity of each individual constituent and τ the associated local characteristic time. The characteristic mean unit of action[2] per constituent can be defined as

$$\alpha \cong mv^2\tau \,, \tag{9}$$

or, alternatively as

$$\alpha \cong rF(r)\tau \,. \tag{10}$$

The second expression is preferable if it is difficult to determine the mass and the characteristic velocity of the particle (*i.e.* to evaluate the characteristic kinetic energy). Eq.(10) defines an impulsive unit of action associated to the time variation of the impulse. Eqs.(9) and (10) do not tell us anything about the stability of the system.

[2]The characteristic action per constituent is conceptually different from the interaction action of field theories used in previous section. For this reason, we use now the symbol α instead of $\mathcal{A}$.

A further hypothesis is that if our system is stable, it obeys the virial theorem in the mean (it is worthwhile to stress that any gravitationally bound system, like a galaxy, can be dynamically treated only under this hypothesis [31]). Then the mean potential energy of a particle must be of the same order of magnitude of its mean characteristic kinetic energy. In other words,

$$\mathcal{L} \cong mv^2\,, \tag{11}$$

is the mean characteristic work performed by the system on a single constituent. Since the system is virialized, we can write

$$\mathcal{L} \cong NF(R)R\,, \tag{12}$$

where R is the mean length scale of the system, of the order of magnitude of its global space extension. The mean characteristic velocity per constituent can then be written as

$$v \cong \sqrt{NF(R)Rm^{-1}}\,. \tag{13}$$

On the other hand, considering the global size R of the system and the characteristic global unit of time of the system $\mathcal{T}$ (that is the characteristic time for the evolution of the system as a whole) we can also write[3]

$$v \simeq \frac{R}{\mathcal{T}}\,. \tag{14}$$

By combining the two previous expressions, the characteristic action per constituent can then be written in the form [5]:

$$\alpha \cong m^{1/2}R^{3/2}\sqrt{F(R)}\frac{\sqrt{N}}{\mathcal{T}}\tau\,. \tag{15}$$

Now, following Calogero [4], let us consider τ as the characteristic time associated to the local chaotic component of the motion that each constituent undergoes due to the force $F(r)$ exherted by all other constituents in the system. Such collective chaotic effect can be modeled, according to Calogero, by some time–statistical fluctuation, which can be mathematically expressed as

$$\tau \cong \frac{\mathcal{T}}{\sqrt{N}}\,, \tag{16}$$

Consequently, the characteristic unit of action becomes independent of the number of constituents, as well as of the global and local characteristic unit of time [5]:

$$\alpha \cong m^{1/2}R^{3/2}\sqrt{F(R)}\,. \tag{17}$$

On the other hand, by the fluctuative hypothesis (16), we can write

$$\alpha \cong \epsilon\tau\,, \tag{18}$$

where ϵ is the characteristic energy per particle given by

$$\epsilon \cong \frac{E}{N}\,. \tag{19}$$

[3]For a galaxy, $\mathcal{T}$ is the time after which the system evolves and become stable. It is of the order of 10 Gyr.

E is the total energy of the system and

$$A \cong E\mathcal{T}\,, \tag{20}$$

is the total action. As a consequence, the total action is related to the characteristic action by the relation

$$\alpha \cong N^{-3/2}A\,. \tag{21}$$

Let us now shortly describe an example of computation of the characteristic unit of action for a stable macroscopic system. For a detailed analysis, and a comprehensive treatment of several other macroscopic systems see [5].

Let us consider a stable bunch of charged particles in an a particle accelerator. Confinement and stability are due to the interactions among the constituents and between the constituents and the external electromagnetic fields. The net effect can be schematized by saying that the single charged particle experiences an effective harmonic force (when higher anharmonic contributions can be neglected). The classical law of force is then $F(R) \cong KR$ where K is the effective phenomenological elastic constant. Then, the characteristic unit of action for each beam constituent is

$$\alpha \cong m^{1/2}R^2K^{1/2}\,. \tag{22}$$

Inserting typical numbers (*e.g.* transverse oscillations of proton at HERA), $K \cong 10^{-9}$g sec^{-2}, $R \cong 10^{-5}$cm, $m_p \cong 10^{-24}$g, we obtain $\alpha \cong h$ (the same result holds for electrons but $K \cong 10^{-8}$g sec^{-2}, and we have to take m_e instead of m_p). The same results are obtained considering beam data from the other currently existing accelerators.

The same techniques can be applied to several other bound electromagnetic systems ranging from atoms to molecular clusters, to plasmas in quasi–equilibrium, to Bose–Einstein condensats and to other systems at mascroscopic and mesoscopic scales. In all cases, the computation of the characteristic unit of action yields Planck action constant h [5]. We can say that all these classical and semiclassical systems can be described by classical mechanics plus a suitable classical fluctuation which mimicks the fundamental quantum structure in yielding a characteristic unit of action of order h. In some sense, h is the quantum signature of the system.

IV. THE CHARACTERISTIC UNIT OF ACTION FOR GRAVITATIONALLY BOUND SYSTEMS

The case of gravity is more subtle due to the intrinsic difference of such an interaction with respect to the others. As we have said, the absence of screening mechanisms allows it to act practically at all scales. However, its intrinsic weakness ($\sim 10^{40}$ times weaker than electromagnetic forces) makes it efficient in forming bound structures only if the other interactions can be neglected. On the other hand, at early epochs, gravity was comparable or stronger than the other forces.

As was shown by Calogero [4], a characteristic action involving Newtonian interactions can be easily constructed. If $m = m_p \simeq 10^{-24}$g is the typical mass of the proton (considering nucleons to be the granular constituents of the Universe), $R \simeq 10^{28}$cm can be assumed as the size of the observed Universe, and if we assume the Newton law of force for $F(r)$, we get

$$\alpha \cong G_N^{1/2}m^{3/2}R^{1/2}\,, \tag{23}$$

which yields $\alpha \simeq h$. From this result, Calogero suggests that the origin of quantization could be attributed to the interaction of every particle with the background gravitational force due to all other particles in the Universe. Such a background interaction generates a chaotic component in the motion of each single particle, with a characteristic time $\tau \simeq 10^{-21}$s measuring the time scale of stochasticity

(zitterbewegung). Essentially, Calogero assumed a weak field limit where the total gravitational energy goes as R^{-1} and relativistic corrections are completely neglected. His point of view is completely Machian and gravity is considered as the fundamental interaction.

Eq.(23) was derived starting from (18) where the characteristic time is constructed putting into (16) the age of the Universe and the expected number of baryons as derived by nucleosynthesis [1]. The energy per particle ϵ, as in Eq.(19), is given by dividing the total gravitational energy by the number of constituents. It is clear that a sort of underlying Large Numbers Hypothesis philosophy [7] is assumed and calculations are carried out without taking into account any specific cosmological model.

Comparing Eq.(23) with Eqs. (20) and (21), we have

$$\alpha \cong \left(\frac{M}{N}\right)^{3/2} (G_N R)^{1/2} \ . \tag{24}$$

We will show how to recover this fundamental equation by applying the scheme of Calogero and De Martino *et al.* in the context of the theories of induced gravity. We have seen in Sec.2, that quantum field theory asks for a "fundamental quantum mechanical nature" of the Universe [32] capable of inducing the today observed gravitational and cosmological constants (without specific assumptions on the initial conditions) just as a consequence of its dynamical behavior. This argument is supported also by quantum cosmology [33],[34].

This point of view is radically different from that of Calogero since the fundamental quantum nature should be recognized at any scale. Microscopic and macroscopic systems, galaxies, clusters, and other large scale structures should show such a quantum signature.

Microscopic systems naturally exhibit quantum signature since h is present in any physical quantity connected with them. The characteristic action for an atom is trivially h. For mesoscopic and macroscopic scales, quantum coherent phenomena are the evidence that quantum mechanics acts also at this level.

In cosmology, the result by Calogero shows that the whole Universe, considered as a gravitationally bound system, shows a quantum signature. The questions now are: Is this true for any gravitationally bound system? Does the gravitational coupling play a role in this picture? Can the absence of screening effects be connected to the fact that quantum gravity fluctuations act at all scales?

As starting point, let us consider again the result (23) by Calogero. At astrophysical and cosmological large scales, the granular constituents of a system can be considered the stars or the galaxies and not a simply inchoerent distribution of baryons[4]. As we have said, stars (and planets) are not properly simple gravitationally bound systems since electromagnetic and nuclear interactions contribute (with gravity) to the stability of the system. Then gravity loses its preminent role to bind a star contrary to what happens, for example, in a galaxy. For this reason, stars can be considered the granular units of the universe. A globular cluster ($\sim 10^6$ stars), a galaxy ($\sim 10^{11}$ stars), a cluster of galaxies ($\sim 10^{13}$ stars), or a supercluster of galaxies ($\sim 10^{17}$ stars) are typical systems completely bound by gravity where the other interactions are negligible, and whose elementary constituents are stars (However, there is an important difference between the globular clusters and the other systems: The former can be considered collisional systems while the other are collisionless systems [31]. This fact changes completely the dynamical treatment of the two classes of objects).

We have that a typical Main Sequence star has a mass

$$M_s \simeq 1 M_\odot = 1.99 \times 10^{33}\mathrm{g} \cong 1.19 \times 10^{57}\mathrm{protons}\,, \tag{25}$$

[4]We are assuming that the main part of the mass in the Universe is clustered in stellar–like aggregates; also the so–called MACHOs (Massive Astrophysical Compact Halo Objects), recently found by microlensing technique[35], have masses of the order of solar mass.

and then

$$m_p = m = \frac{M_s}{N} \cong 10^{-24}\text{g}\,, \tag{26}$$

which is the mass in Eq.(23). Now let us take into account a typical galaxy. Its mass is

$$M_g \simeq 10^{11} M_\odot \cong 1.19 \times 10^{68}\text{protons}\,, \tag{27}$$

so that Eq.(26) is again recovered. The situation, as obvious, is exactly the same for globular clusters, clusters and superclusters of galaxies. Then the ratio

$$\frac{M}{N} \simeq 10^{-24}\text{g}\,, \tag{28}$$

is the same for any gravitationally bound system.

Let us now compute the characteristic unit of action considering, at first, the gravitational coupling simply as the Newton constant G_N. A typical galaxy like the Milky Way has a linear size of the order $R \simeq 30$kpc (a parsec is approximatively 1pc=3.1×10^{18}cm), a mass of the order $M_g \simeq 10^{11} M_\odot$. As previously seen, the number of baryons is $N \simeq 10^{68}$. Introducing these numbers into Eq.(24), we get

$$\alpha \simeq 10^{-28}\text{erg s}\,. \tag{29}$$

Considering now a typical cluster of galaxies with linear size $R \simeq 10$Mpc and $M_c \simeq 10^{13} M_\odot$, we obtain

$$\alpha \simeq 10^{-27.5}\ \text{erg s}\,. \tag{30}$$

Finally, at supercluster scales, *i.e.* $R \simeq 100$Mpc and $M_{sc} \simeq 10^{17} M_\odot$, we obtain

$$\alpha \simeq 10^{-27}\text{erg s} \simeq h\,, \tag{31}$$

that is approximatively the Planck constant value. All these values must be taken with an error of an order of magnitude.

It is interesting to observe that the result of Calogero for the whole Universe is better recovered the larger is the structure considered. However, we have to take care of the large uncertainties with which cosmological quantities are known. In fact, the order of magnitude of the Hubble radius is $R \simeq 3 \times 10^3 \tilde{h}^{-1}$Mpc with $0.4 \leq \tilde{h} \leq 1$ depending on the value of the Hubble constant H_0 which, actually, is very controversial [36].

In any case, ranging from galaxies to very large scale structures, as superclusters, any gravitationally bound system has an associated characteristic unit of action per constituent whose value is very close to the Planck constant. For very large structures, the characteristic action completely coincides with h.

We now wish to show that, assuming a running gravitational coupling which decreases with distance (as supposed by Dirac since cosmological times and distances are related), we can show that any gravitationally bound system has a characteristic action coinciding with h.

To obtain such a result, we have to replace Eq.(24) with

$$\alpha = \left(\frac{M}{N}\right)^{3/2} \left[G(R)R\right]^{1/2}\,, \tag{32}$$

which is in the same line of Eq.(17). The ratio M/N is always of the order $\sim 10^{-24}$g, the other term must be

$$\left[G(R)R\right]^{1/2} \simeq 10^{9}\text{cm}^2\text{s}^{-1}\text{g}^{-1/2}\,. \tag{33}$$

For very large scales we must have that $G(R) \to G_N$ which, besides, has to coincides with the value measured inside the Solar System and at laboratory level [37]. The product $G(R)R$ is the strength of the gravitational interaction which is scale–dependent [24].

Reproducing h (at any scale) could be considered the quantum signature for gravitationally bound systems at any cosmological scales. Exploiting the laws of variation obtained by the theories of induced gravity in Sec.2, it is easy to reproduce the constraint (33). For example, by Eq.(5), if we choose the set of numbers $\eta = -11$, $r_0 = 10$kpc and $\chi = 1/30$, we recover $\alpha \simeq h$ for a typical galaxy of size $R \simeq 30$kpc. It is worthwhile to note that the parameter $r_0 \simeq 10$kpc is often used to reproduce the flat rotation curves of spiral galaxies [24],[28]. It could be recovered also taking into consideration the emittance which is a scale of length (or of "temperature") associated to a correlated system [5]. Such a quantity is defined as

$$\mathcal{E} \simeq \lambda_c \sqrt{N}\,, \tag{34}$$

where $\lambda_c = h/mc$ is the Compton length. It is connected to the characteristic action [5]. If we consider the Compton length of the proton and the number N of protons in a galaxy, we obtain $\mathcal{E} \simeq 10$kpc, that is the emittance is connected to the typical scale length of the galaxy. In other worlds, the quantum parameter λ_c and the number of microscopic constituents N determine the astrophysical size $\mathcal{E}$.

Similar results, can be achieved also using exponential laws as (7) or (8). However, the parameters (and, in some sense, the right law) depend on the scale which we are considering and only observations can fix exactly the model.

The situation is very close to that of ordinary quantum systems: Given a set of quantum numbers, we obtain a stable state. In this case, given a set of gravitational–quantum numbers, we obtain a gravitationally bound stable system. This could be the simple explaination why gravitationally bound systems do not form at any scale.

As a further remark, we can say that also the problem of dark matter is not so dramatic if one agrees with this picture. The large amount of gravitating material which people observe at any scale is only due to the fact that Newton law is always used without considering the change of the strength of the gravitational interaction.

A last point of controversy is if gravity increases or decreases with the scale (see for example [27],[38],[39]). Actually, the renormalized one–loop quantum gravity models forecast both the options depending on the parameters of the renormalization group equations [25],[26]. Only astrophysical observations and fine laboratory experiments on the variation of $G(R)$ will decide what is the real situation [37].

V. CONCLUSIONS

In this paper, we argued that any large scale bound system, where gravity is the overall interaction among the components, has a characteristic unit of action which coincides, in order of magnitude, with the Planck constant. The result is achieved by asking for the variation of gravitational coupling with scale. This issue is in agreement with quantum gravity models and with the fact that a screening mechanism is absent in gravity.

Also if a similar result for the whole Universe was achieved by Calogero, his point of view is different since, in his picture, gravity is the fundamental interaction that, in a Machian way, produced a sort of stochastic quantization. In our case, gravity is induced by quantum field theory and the Universe "at all scales" has a fundamental quantum mechanical nature. However, both approaches ask for a quantum signature which can be achieved at large scales and, also if we are considering a variable gravitational coupling, the effects of such a variation are small (They are confined into one or two order of magnitude. Besides, the expected variation of Newton constant into the solar system is estimated to be $\dot{G}/G \sim 10^{-11}$ years^{-1} which is very small [37]).

If this approach is fundamentally sensible, questions like the formation of galaxies, the dark matter problem, and the role of gravity as a fundamental interaction have to be deeply reconsidered.

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